

Space Interferometry Mission System Testbed-3: External Metrology Inversion^{1, 2}

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Abstract - The Space Interferometry Mission's System testbed-3 has recently integrated its precision support structure and spacecraft bus, or 'backpack', on a pseudo free-free 0.5 Hz passive isolation system. The precision support structure holds a 3-baseline stellar interferometer instrument. The architecture of the instrument is based on the current SIM Flight System design, and its main purpose is to demonstrate nanometer class fringe stabilization using the path length feed forward technique. This paper briefly describes the nanometer-class metrology system used in this testbed to estimate the length and orientation of the science baseline vector, which cannot be measured directly. The focus is on the mathematical inversion problem that results and its solution.

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1. INTRODUCTION

The space interferometry mission, SIM, System Testbed-3, STB3, has been integrated at the Jet Propulsion Laboratory. The testbed instrument is designed to have the functionality of the SIM Flight System, and is charged with demonstrating nanometer-class fringe stability using the Path Length Feed Forward Technique, PFF. Synthesis of PFF commands is realized with the use of two guide stars (to measure the rigid body attitude of the instrument

in space), and an external metrology system to measure the relative motion of the instrument fiducials [1]. A brief description of the metrology system is discussed in section 2. Derivation of the inversion problem that results and its solution are discussed in Section 3. Section 4 discusses simulations run to show convergence of the solution, and the sensitivity matrix.

2. METROLOGY SYSTEM DESCRIPTION

The metrology system consists of 14 metrology beam launchers, which measure the relative motion of 6 fiducials in the testbed. There are a total of 15 possible measurements that can be made between all the fiducials. The measurement that corresponds to the relative motion of the science interferometer fiducials cannot be made directly, but can be estimated from the other 14 measurements. In addition, the relative orientation of the science baseline vector connecting its fiducials is obtained. Figure 1 shows the STB3 setup, which depicts the pseudo star system, the three baseline astrometric interferometer, the starlight paths, and the 14 external metrology beams. The external metrology beam launchers are not shown for clarity..

3. TRUSS EQUATIONS

Each metrology gauge measures the relative displacements between its fiducials. The measurements are relative, so in order to estimate the motion of the science fiducials, the relative locations of all 6 fiducials have to be obtained using only an initial survey of their locations and the measured changes in gauge readings given by the metrology system.

First, a relationship between fiducial motion and gauge measurements is needed. Consider a pair of fiducials and a change in their location [2]. This change generates a change in gauge reading. Figure 1 shows the two fiducials in their original and new locations. The following definitions apply to figure 1:

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² IEEEAC paper #1117, Version 5, Updated January 15, 2004

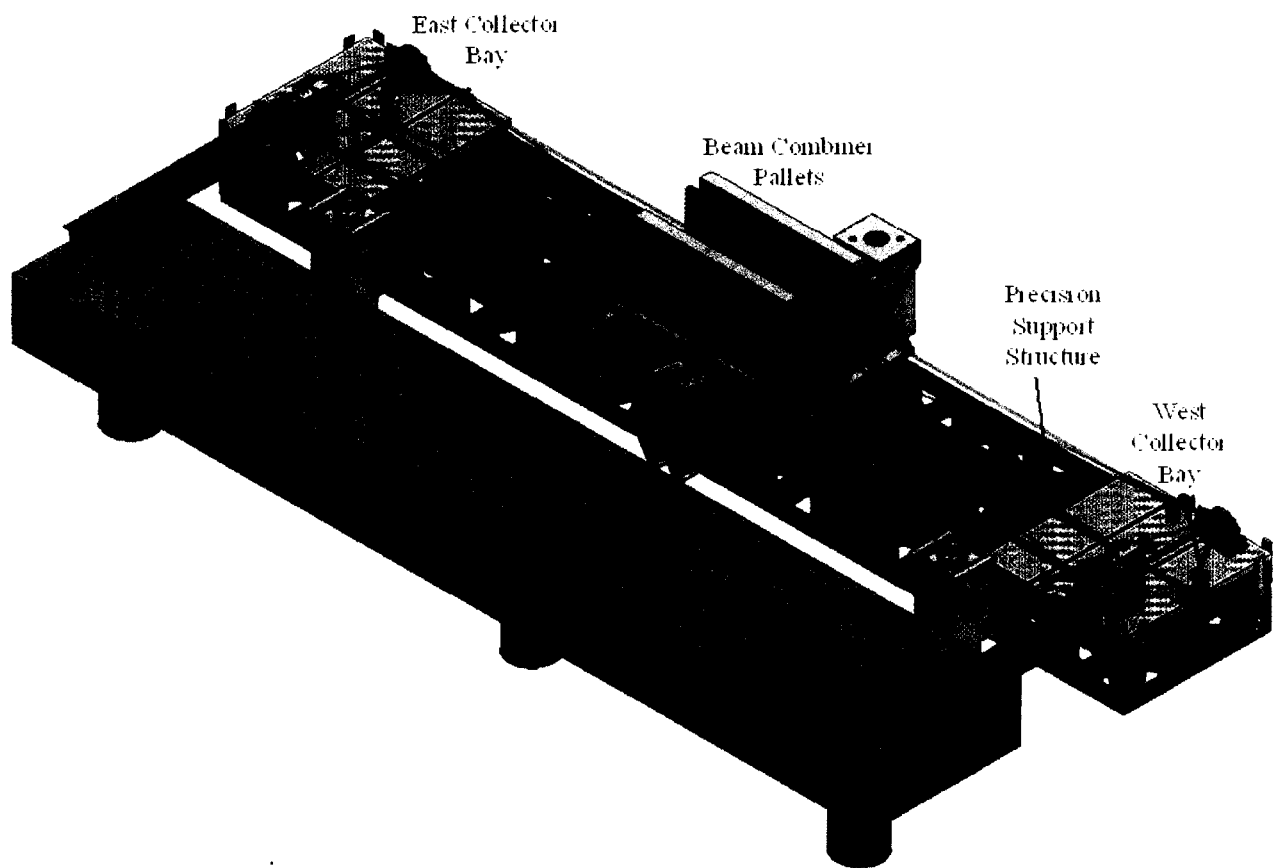


Figure 1 SIM's System Testbed 3. Shown in the figure are the pseudo star system and the 3-baseline astrometric interferometer instrument. The starlight path for each baseline and the 14 external metrology beams are also depicted.

F_1 : Location of Fiducial 1
 F'_1 : Location of Fiducial 1 after a small change in position
 F_2 : Location of Fiducial 2
 F'_2 : Location of Fiducial 2 after a small change in position
 \vec{x}_1 : vector to F_1
 \vec{x}_2 : vector to F_2
 $\vec{x}_{1,2}$: vector from F_1 to F_2
 $\Delta\vec{x}_1$: change in fiducial vector 1
 $\Delta\vec{x}_2$: change in fiducial vector 2

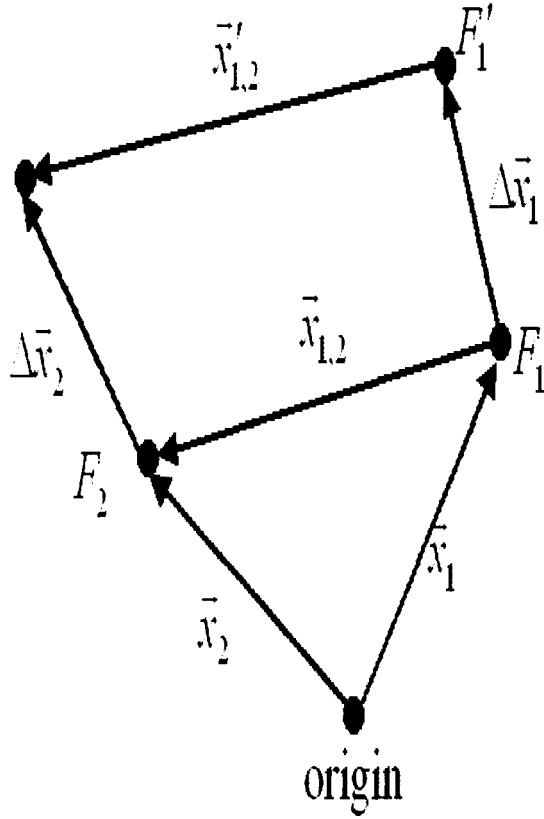


Figure 2 Relationship between fiducials and gauge measurements

The following relationships follow from inspection of Figure 2,

$$\begin{aligned}
 \Delta|\vec{x}_{1,2}| &= \text{Gauge Reading} \\
 &= |\vec{x}'_{1,2}| - |\vec{x}_{1,2}| \\
 &\equiv \frac{\vec{x}_{1,2}}{|\vec{x}_{1,2}|} \cdot \Delta\vec{x}_{1,2}
 \end{aligned}$$

where

$$\Delta\vec{x}_{1,2} = \Delta\vec{x}_2 - \Delta\vec{x}_1$$

then,

$$\Delta|\vec{x}_{1,2}| = \frac{\vec{x}_{1,2}}{|\vec{x}_{1,2}|} \cdot (\Delta\vec{x}_2 - \Delta\vec{x}_1)$$

which can be re-written in matrix form:

$$\Delta|\vec{x}_{1,2}| = \begin{pmatrix} -\frac{\vec{x}_{1,2}}{|\vec{x}_{1,2}|} & \frac{\vec{x}_{1,2}}{|\vec{x}_{1,2}|} \end{pmatrix} \begin{Bmatrix} \Delta\vec{x}_1 \\ \Delta\vec{x}_2 \end{Bmatrix} \quad (1)$$

Equation 1 represents a linear relationship between a gauge reading and changes in fiducial locations. In order to reduce the size of ensuing equations, Equation 1 is re-written in terms of the fiducial to fiducial unit vector, $\hat{u}_{1,2}$.

$$\Delta|\vec{x}_{1,2}| = \begin{pmatrix} -\hat{u}_{1,2} & \hat{u}_{1,2} \end{pmatrix} \begin{Bmatrix} \Delta\vec{x}_1 \\ \Delta\vec{x}_2 \end{Bmatrix} \quad (1a)$$

where

$$\hat{u}_{i,j} = \frac{\vec{x}_{i,j}}{|\vec{x}_{i,j}|}, \quad \hat{u}_{i,j} \in \mathbb{R}^3$$

represents the unit vector from the i^{th} to the j^{th} fiducial.

This equation can be extended to include all 6 fiducials and all 14 beam-launchers. Define r as the set of gauge measurements, and v the set of fiducial locations:

$$r = \{ \Delta|\vec{x}_{i,j}| \}, \quad j > i, \quad i = 1 \rightarrow 5, \quad j = 2 \rightarrow 6$$

where

$$\Delta|\vec{x}_{1,4}| \notin r$$

and

$$v = \{ \Delta\vec{x}_i \}, \quad i = 1 \rightarrow 6$$

These definitions can be used to extend Equation 1 as follows,

$$r = Av \quad (2)$$

where the rows of the matrix A (14 rows by 18 columns) define the relationship between each gauge measurement and its pair of fiducials. The matrix A is defined below,

$$A = \begin{bmatrix} -\hat{u}_{1,2} & \hat{u}_{1,2} & 0 & 0 & 0 & 0 \\ -\hat{u}_{1,3} & 0 & \hat{u}_{1,3} & 0 & 0 & 0 \\ -\hat{u}_{1,5} & 0 & 0 & 0 & \hat{u}_{1,5} & 0 \\ -\hat{u}_{1,6} & 0 & 0 & 0 & 0 & \hat{u}_{1,6} \\ 0 & -\hat{u}_{2,3} & \hat{u}_{2,3} & 0 & 0 & 0 \\ 0 & -\hat{u}_{2,4} & 0 & \hat{u}_{2,4} & 0 & 0 \\ 0 & -\hat{u}_{2,5} & 0 & 0 & \hat{u}_{2,5} & 0 \\ 0 & -\hat{u}_{2,6} & 0 & 0 & 0 & \hat{u}_{2,6} \\ 0 & 0 & -\hat{u}_{3,4} & \hat{u}_{3,4} & 0 & 0 \\ 0 & 0 & -\hat{u}_{3,5} & 0 & \hat{u}_{3,5} & 0 \\ 0 & 0 & -\hat{u}_{3,6} & 0 & 0 & \hat{u}_{3,6} \\ 0 & 0 & 0 & -\hat{u}_{4,5} & \hat{u}_{4,5} & 0 \\ 0 & 0 & 0 & -\hat{u}_{4,6} & 0 & \hat{u}_{4,6} \\ 0 & 0 & 0 & 0 & -\hat{u}_{5,6} & \hat{u}_{5,6} \end{bmatrix}$$

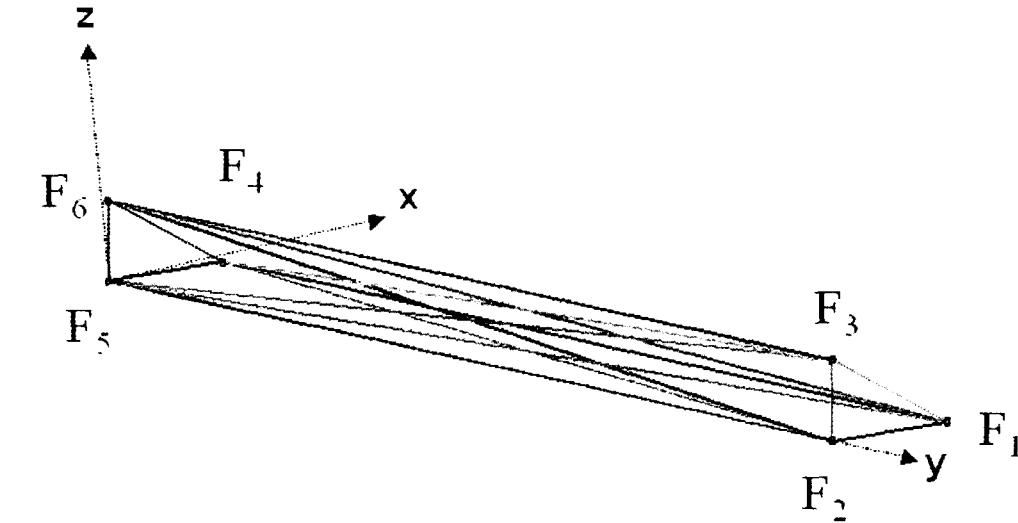
Equation 2 has no solution in its present form, since the columns of A are not linearly independent for the case

where $m > n$ (i.e., A has more columns than rows) [3]. This is remedied with the definition of an appropriate coordinate system for the fiducials, and the realization that in this coordinate system 6 of the 18 fiducial degrees of freedom can be fixed.

Figure 3 shows the geometry of the truss and all 6 fiducials, complete with a convenient choice of coordinate system for STB3. In this coordinate system, Fiducial 5 (East guide fiducial) is chosen to be at the origin of the coordinate system. Hence, since our gauge measurements are relative to the 6 gauges, Fiducial 5 can be assumed to be stationary. Moreover, this fiducial becomes the tie between the metrology truss and the rigid body motions of the spacecraft holding the instrument.

Figure 3 also shows fiducial 2 as being on y-axis of the coordinate system, and fiducial 1 as being on the x-y plane. This definition of the coordinate system fixes 6 out of the 18 fiducial degrees of freedom. Given that each $\Delta \vec{x}_i \in \mathbb{R}^3$, the rows of A corresponding to

$$\Delta x_{1,z}, \Delta x_{2,x}, \Delta x_{2,z}, \Delta x_{5,x}, \Delta x_{5,y}, \Delta x_{5,z}$$



- »F1 = Science Sid Fiducial – West → Coincides with X-Y plane
- »F2 = Guides Fiducial – West → coincides with Y axis
- »F3 = Fake Science Fiducial – West
- »F4 = Science Sid Fiducial – East
- »F5 = Guides Fiducial – East → Origin of Truss Coordinate System
- »F6 = Fake Science Fiducial – East

Figure 3 Coordinate system for metrology truss

where

$$\Delta \vec{x}_i \equiv \left\{ \Delta x_{i,x} \hat{i} \quad \Delta x_{i,y} \hat{j} \quad \Delta x_{i,z} \hat{k} \right\}$$

can be eliminated from A . So, the number of unknown fiducial-degrees-of-freedom is reduced to 12, whereas the number of measurements remains at 14. Equation 2 can now be solved,

$$v = A^+ r \quad (3)$$

where A^+ is the pseudo inverse of A whose existence is guaranteed for the case where the columns of A are linearly independent. Inspection of matrix A shows that its 12 columns are linearly independent.

The degree of goodness of the solution offered in Equation 3 has to be considered in the presence of noisy measurements (i.e., r is a noisy set of gauge measurements). So, assume \hat{v} to be a noisy estimate, and proceed to reduce its error variance with an iterative regularization scheme, where a quadratic function of \hat{v} , $f(\hat{v})$, is minimized. Without stating details, we chose a steepest descent method, which yields the following iterative optimization of the estimate of \hat{v} ,

$$\hat{v}_{n+1} = \hat{v}_n - 2A^+ \hat{e}_n \quad (4)$$

where

$$\hat{e}_n = r - A\hat{v}_n$$

Single step convergence of Equation 4 is guaranteed by the convergence theorem [3]. So, once a minimum variance estimate of the positions of all fiducials in the truss has been obtained, the science baseline length and orientation relative to the truss coordinate system is trivial. In STB3 an algorithm for Equation 4 is implemented in the real time control system, which generates estimates of the fiducial locations at a rate of 1000 times per second. These estimates are then used in the synthesis of the path length feed forward command for the science baseline.

4. PRELIMINARY RESULTS

The definition of matrix A in equation 2 implied a priori knowledge of the unit vectors from the i^{th} to the j^{th} fiducial. This is accomplished with an initial survey of the truss, which, for STB3, we assume can generate 500 micro-meter class estimates of the initial fiducial locations (SIM will actually have a 3 micrometer class absolute metrology system to obtain these measurements every time a new tile observation is started [4], but this level of precision is only needed for picometer class regularization of the truss solutions.) Monte-Carlo simulations of the truss solution for STB3 using this

assumption about the initial truss knowledge, but assuming noiseless gauge measurements, yield a 0.1-nanometer average estimate error for the science baseline length estimate ($\sigma = 1.5$ nm). When gauge noise is included in the simulations (zero mean, 10 nm RMS – based on preliminary measurements) the mean of the estimate error goes up to 0.6 nm ($\sigma = 11.5$ nm). For these simulations the length of the truss was taken as 8 meters, its height was taken as 0.5 meters, and its width was taken as 1 meter (see Figure 1)

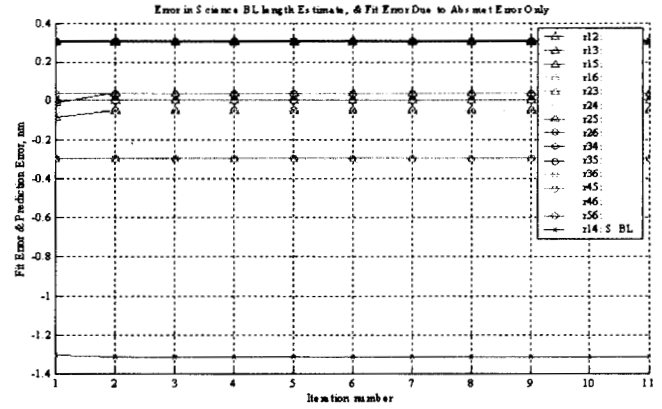


Figure 4 Typical truss solution estimate and fit error convergence

Figure 4 shows a typical simulated convergence plot for the fit error and science baseline estimate error. Note that a single iteration is sufficient to converge as was stated in section 3.

Figure 5 shows the sensitivity matrix A^+ plotted in a 3-dimensional graph to show the error sensitivity of each fiducial coordinate, as function of each external metrology measurement. Note that fiducials 4 (z coordinate) and 6 (x coordinate) are most sensitive to gauge measurements. In general the sensitivity matrix appears to be well conditioned, while indicating greater sensitivity due to measurements made along the longer beam paths (i.e., measurements across the long axis of the precision support structure).

5. SUMMARY

The External metrology system for SIM's System Testbed 3 was described here, as well as the method of solution for the metrology truss equations. Simulations show that a 500 mm class initial survey of the testbed would be sufficient to yield a nanometer-class accuracy estimate of the science baseline change in length in the presence of noisy gauge readings. Not discussed in this paper is the error in the metrology system due to index of refraction fluctuations in the laboratory. Atmospheric effects limiting the performance of the system, plus simulations using a detailed Finite Element Model of the testbed are currently under way and will be reported on later

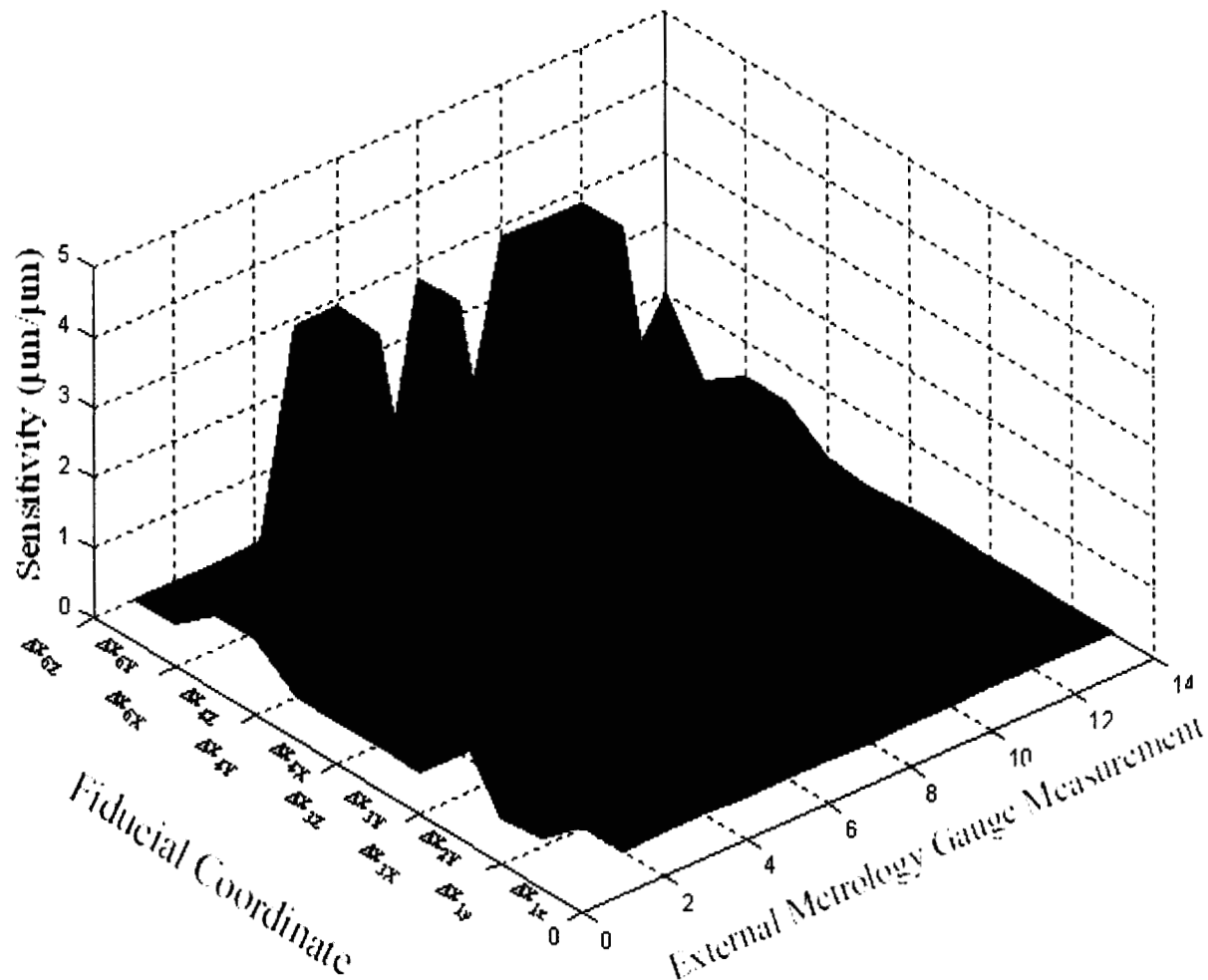


Figure 5 Sensitivity of estimate of fiducial coordinate due to external metrology measurements

publication. Actual system performance will be assessed in the first half of 2004.

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7. REFERENCES

- [1] R. Goullioud, O.S. Alvarez-Salazar, and B. Nemati, "Path feed forward performance of an astrometric 3-BL interferometer testbed (STB-3)," *SPIE conference on Interferometry in Space*, vol. 4852, 2002
- [2] Private communication with B. Nemati
- [3] Luenberger, D. G., *Optimization by Vector Space Methods*, John Wiley & Sons, New York, 1969

[4] J. Marr, "Space Interferometry Mission (SIM) overview and current status," *SPIE conference on Interferometry in Space*, vol. 4852, 2002.

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